Projectile Motion (Non-Horizontal Projection)

The following applies to situations in which a projectile returns to its launch height.

(I call these "level field" problems)

Such an object has a vertical displacement of zero, so in the y-direction:

$$\overrightarrow{\Delta d} = 0$$

$$\overrightarrow{\Delta d} = \vec{v}_1 \Delta t + \frac{1}{2} \vec{a} \Delta t^2$$

$$0 = v_1 \sin\theta \Delta t + \frac{1}{2}(-g)\Delta t^2$$

$$\Delta t = \frac{2 v_1 \sin\theta}{g}$$

This is called the ***"time of flight"*** for the projectile, the time taken to return to launch height.

To find the horizontal distance the projectile travels, called the ***"range"***,

$$\overrightarrow{\Delta d}_x = \vec{v}_x \Delta t$$

(Since acceleration is zero in the x-direction)

$$\Delta d_x = v_1 \cos\theta \, \Delta t$$

$$\Delta d_x = v_1 \cos\theta \left(\frac{2 v_1 \sin\theta}{g} \right)$$

$$\Delta d_x = \frac{2 v_1^2 \sin\theta \cos\theta}{g}$$

$$\Delta d_x = \frac{v_1^2 \sin 2\theta}{g}$$

So to maximize the range of the projectile, we must maximize

$$\sin 2\theta$$

Thus the launch angle must be 45º to maximize range.

1. Iggy punts a rugby ball with a launch speed of 22 m/s at an angle of 38º above the horizontal. Determine the

a) "Hang time" of the ball.
b) the range of the kick.

(ignore the height of the ball when kicked) a)

$$\Delta t = \frac{2 v_1 \sin\theta}{g}$$

$$\Delta t = \frac{2(22)\sin 38^o}{9.8}$$

$$\Delta t = 2.76\,s$$

b)

$$\Delta d_x = \frac{v_1^2 \sin 2\theta}{g}$$

$$\Delta d_x = \frac{(22)^2 \sin 2(38^o)}{9.8}$$

$$\Delta d_x = 48\, m$$

2. Ringo the human cannonball is fired from a 32 m high cliff with a launch speed of 28 m/s at an angle of 48º above the horizontal. Determine

a) his flight time.
b) the velocity with which he hits the water below.