SPH4U Sample Test - Kinematics

Modified True/False
Indicate whether the sentence or statement is true or false. If false, change the identified word or phrase to make the sentence or statement true.

____ 1. The slopes of position-time, velocity-time, and acceleration-time graphs represent velocity, acceleration, and *displacement*, respectively. _____

____ 2. Two siblings leave their house and walk to the same school taking different routes. One of the routes is farther, but the siblings take the same time to reach the school. It can be said that the two siblings have the *same average velocity* but different average speeds. _____

____ 3. A car drives with constant speed around a corner. If it enters the corner travelling west and leaves travelling south, its acceleration through the turn is directed *south-west*. _____

____ 4. An object is thrown vertically upward. At the top of its flight, when its velocity is momentarily zero, its acceleration is *zero*. _____

____ 5. If air resistance is negligible, the acceleration of all projectiles is *exactly the same*. _____

____ 6. To achieve maximum range across a horizontal surface, the angle at which a projectile must be launched is *90°*. _____

____ 7. Provided that a boat always points *perpendicular to the current* in a river, the time it takes the boat to cross is independent of the strength of the current. _____

____ 8. A boat points directly across a river and gets carried 15° downstream by the current. To land at a position directly across the river from its starting point, the boat must point at an angle *slightly less than 15°* upstream. _____

Multiple Choice
Identify the letter of the choice that best completes the statement or answers the question.

____ 9. The slope of a line drawn tangent to a curved position-time graph represents
a. displacement
b. instantaneous velocity
c. average velocity
d. acceleration
e. distance

____ 10. An object is thrown vertically upward at 18 m/s from a window and hits the ground 1.6 s later. What is the height of the window above the ground? (Air resistance is negligible.)
a. 3.7 m
b. 16 m
c. 21 m
d. 37 m
e. 41 m

____ 11. A jogger is running at 4.2 m/s when she begins to accelerate uniformly. If she runs a distance of 14 m in the next 3.0 s, what is her new speed?
a. 17 m/s
b. 14 m/s
c. 7.7 m/s
d. 5.1 m/s
e. 4.9 m/s

____ 12. What distance does an object travel during a period of uniform acceleration (a = 2.5 m/s^2) when its speed changes from 35 m/s to 45 m/s?
a. 6.5×10^2 m
d. 32 m

b. 3.2×10^2 m
c. 1.6×10^2 m
e. 2.0 m

____ 13. The acceleration due to gravity on Earth
a. is the same at all locations on the surface of Earth
b. is greater for heavier objects
c. is greater at the equator and less at the poles
d. is the same at any two locations provided that the distance to the centre of Earth at those locations is the same
e. varies slightly with latitude

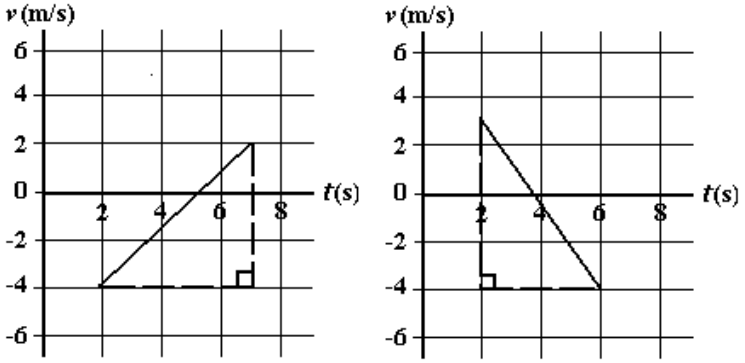
____ 14. For a javelin thrower to maximize her throwing distance, she should release the javelin at an angle
a. of 45°
b. of less than 45°
c. of greater than 45°
d. dependent on the javelin's speed upon release
e. dependent on the thrower's strength

____ 15. A boat always points directly at the opposite shore while crossing a river. The time it will take to cross will be
a. less if the current is stronger
b. greater if the current is stronger
c. the same regardless of the current
d. dependent on the strength of the current
e. impossible to predict without more information

____ 16. A plane must fly to a destination located due north from its departure point. A gentle wind is blowing from the south-west. What direction must the plane point to reach its destination?
a. north
b. north-west
c. north-east
d. west
e. east

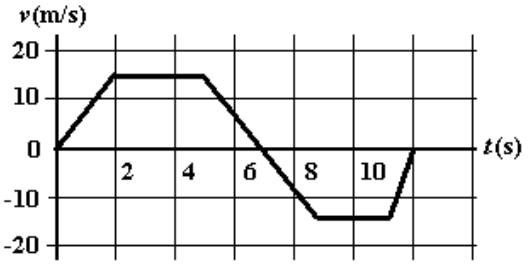
Short Answer

17. These graphs represent the acceleration of two objects. Determine the acceleration of each object.



18. A steel ball, starting from rest, rolls down one slope and up another. It takes 2.5 s to reach the bottom of the first slope, at which point its speed is 5.0 m/s. If the magnitude of the acceleration on the second slope is exactly one-half that on the first slope, how long will it take for the ball to come to a stop on the second slope?

19. From the *v*-*t* graph below, draw the corresponding *d*-*t* graph and *a*-*t* graph. (Assume that $d_1 = 0$.)



Problem

20. A baseball is hit by a bat and given a velocity of 40.0 m/s at an angle of 30.0° above the horizontal. The height of the ball above the ground upon impact with the bat is 1.0 m.
    (a) What maximum height above the ground does the ball reach?
    (b) A fielder is 110.0 m from home plate when the ball is hit and the ball's trajectory is directly at him. If he begins running at the moment the ball is hit and catches the ball when it is still 3.0 m above the ground, how long does he run before catching the ball?
    (c) How fast (average speed) does he have to run in order to catch the ball?

21. A man walks 600 m [E47°N], then 500 m [N38°W], then 300 m [W29°S], and finally 400 m [S13°E]. Find his resultant displacement.

22. A dog walks at 1.6 m/s on the deck of a boat that is travelling north at 7.6 m/s with respect to the water.
    (a) What is the velocity of the dog with respect to the water if it walks towards the bow?
    (b) What is the velocity of the dog with respect to the water if it walks towards the stern?
    (c) What is the velocity of the dog with respect to the water if it walks towards the east rail, at right angles to the boat's keel?

23. A clock has a second hand that is 12 cm long. Find each of the following.
    (a) the average speed of the tip of the second hand
    (b) its instantaneous velocity as it passes the 6 and the 9 on the clock face
    (c) its average velocity in moving from the 3 to the 12 on the clock face
    Note: the circumference of a circle is $2\pi r$.

SPH4U Sample Test - Kinematics
Answer Section

MODIFIED TRUE/FALSE

1. ANS: F, jerk REF: K/U OBJ: 1.2
   LOC: FM1.02
2. ANS: T REF: K/U OBJ: 1.1
   LOC: FM1.02
3. ANS: F, south-east REF: K/U OBJ: 1.2
   LOC: FM1.05
4. ANS: F, 9.8 m/s^2 [down] REF: K/U OBJ: 1.3
   LOC: FM1.05
5. ANS: T REF: K/U OBJ: 1.4
   LOC: FM1.03
6. ANS: F, 45° REF: K/U OBJ: 1.4
   LOC: FM1.03
7. ANS: T REF: K/U OBJ: 1.5
   LOC: FM1.02
8. ANS: F, slightly greater than 15° REF: K/U OBJ: 1.5
   LOC: FM1.02

MULTIPLE CHOICE

9. ANS: B REF: K/U OBJ: 1.1 LOC: FM1.02
10. ANS: B REF: K/U OBJ: 1.3 LOC: FM1.02
11. ANS: D REF: K/U OBJ: 1.2 LOC: FM1.02
12. ANS: C REF: K/U OBJ: 1.2 LOC: FM1.02
13. ANS: E REF: K/U OBJ: 1.3 LOC: FM1.03
14. ANS: A REF: K/U OBJ: 1.4 LOC: FM1.03
15. ANS: C REF: K/U OBJ: 1.5 LOC: FM1.02
16. ANS: B REF: K/U OBJ: 1.5 LOC: FM1.02

SHORT ANSWER

17. ANS:
    acceleration = slope of *v*-*t* graph

$$a = \frac{\Delta v}{\Delta t}$$

$$= \frac{v_2 - v_1}{t_2 - t_1}$$

$$= \frac{2.0 \text{ m/s} - (-4.0 \text{ m/s})}{7.0 \text{ s} - 2.0 \text{ s}}$$

$$= \frac{6.0 \text{ m/s}}{5.0 \text{ s}}$$

$$= 1.2 \text{ m/s}^2$$

acceleration = slope of *v*-*t* graph

$$a = \frac{\Delta v}{\Delta t}$$

$$= \frac{v_2 - v_1}{t_2 - t_1}$$

$$= \frac{-4.0 \text{ m/s} - 3.0 \text{ m/s}}{6.0 \text{ s} - 2.0 \text{ s}}$$

$$= \frac{-7.0 \text{ m/s}}{4.0 \text{ s}}$$

$$= 1.8 \text{ m/s}^2$$

REF: K/U OBJ: 1.2 LOC: FM1.02 KEY: FOP 2.8, p.61
MSC: SP

18. ANS:

$$a_1 = \frac{v_2 - v_1}{a}$$

$$= \frac{5.0 \text{ m/s} - 0 \text{ m/s}}{2.5 \text{ s}}$$

$$= 2.0 \text{ m/s}^2$$

But $a_2 = \frac{1}{2} a_1 = 1.0 \text{ m/s}^2$

Since the ball will decelerate on the second slope,

$a_2 = -1.0\ \text{m/s}^2$

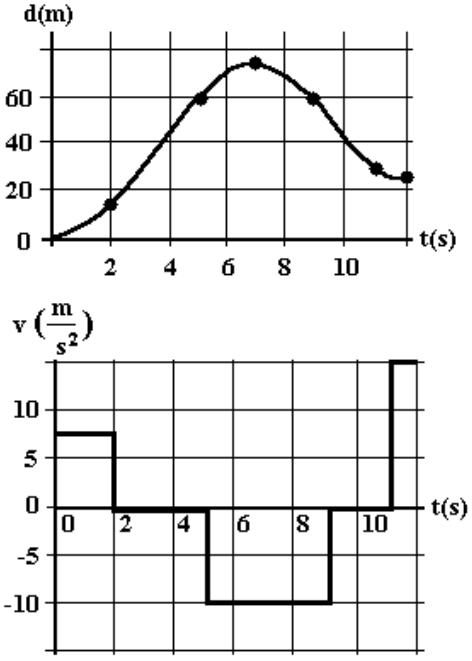
$$\Delta t = \frac{v_2 - v_1}{a}$$

$$= \frac{0\ \text{m/s} - 5.0\ \text{m/s}}{-1.0\ \text{m/s}^2}$$

$$= 5.0\ s$$

REF: K/U OBJ: 1.2 LOC: FM1.02 KEY: FOP 2.9, p.65
MSC: P

19. ANS:



REF: K/U, C OBJ: 1.2 LOC: FM1.02 KEY: FOP 2.12, p.69
MSC: P

PROBLEM

20. ANS:
(a)
At maximum height: vertical component of velocity is zero:
let "up" be (–) and "down" be (+)
$v_1 = -40.0\ \text{m/s}(\sin 30.0^\circ) = -20.0\ \text{m/s}$
$a = 9.8\ \text{m/s}^2$
$v_2 = 0.0\ \text{m/s}$
$\Delta\ d = ?$

$$\Delta d = \frac{v_2^2 - v_1^2}{2a}$$

$$= \frac{(0.0 \text{ m/s})^2 - (-20.0 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)}$$

$$\Delta d = -20 \text{ m}$$

The ball reaches a maximum height of 21 m above the ground. (hit from 1.0 m above the ground)

(b)
Time of flight:
$v_1 = -40.0 \text{ m/s}(\sin 30.0^\circ) = -20.0 \text{ m/s}$
$a = 9.8 \text{ m/s}^2$
$\Delta d = -2.0 \text{ m}$
$\Delta t = ?$

$$\Delta d = v_1 \Delta t + \frac{1}{2} a (\Delta t)^2$$

$$-2.0 = (-20.0)\Delta t + 4.9(\Delta t)^2$$

Solving the quadratic: $\Delta t = 0.10$ s (way up) and 3.98 s (way down)
The fielder must run for 4.0 s in order to catch the ball.

(c)
Horizontal range: $\Delta d = v\Delta t = 40.0 \text{ m/s}(\cos 30^\circ)(3.98 \text{ s}) = 138 \text{ m}$
The fielder must run a distance of: 138 m – 110.0 m = 28 m.

$$v = \frac{\Delta d}{\Delta t}$$

The speed of the fielder:

$$= \frac{28 \text{ m}}{3.98 \text{ s}}$$

$$v = 7.0 \text{ m/s}$$

The fielder must run with an average speed of 7.0 m/s.

REF: K/U OBJ: 1.4 LOC: FM1.03

21. ANS:
Using components in the *x*-*y* plane:

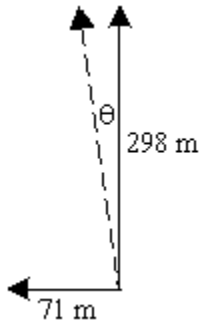
$\Delta\vec{d}_1 = 600\text{ m [E47°N]}$ $\Delta d_{1x} = 600\text{ m } \cos 47° = 409\text{ m}$ $\Delta d_{1y} = 600\text{ m } \sin 47° = 439\text{ m}$

$\Delta\vec{d}_2 = 500\text{ m [N38°N]}$ $\Delta d_{2x} = -500\text{ m } \sin 38° = -308\text{ m}$ $\Delta d_{2y} = 500\text{ m } \cos 38° = 394\text{ m}$

$\Delta\vec{d}_3 = 300\text{ m [W29°S]}$ $\Delta d_{3x} = -300\text{ m } \cos 29° = -262\text{ m}$ $\Delta d_{3y} = -300\text{ m } \sin 29° = -145\text{ m}$

$\Delta\vec{d}_4 = 400\text{ m [S13°E]}$ $\Delta d_{4x} = 400\text{ m } \sin 13° = 90\text{ m}$ $\Delta d_{4y} = -400\text{ m } \cos 13° = -390\text{ m}$

Adding Components: $\Delta d_x = -71\text{ m}$ $\Delta d_y = 298\text{ m}$



$$\left|\Delta\vec{d}\right|^2 = \Delta d_x{}^2 + \Delta d_y{}^2$$

$$= (71\text{ m})^2 + (298\text{ m})^2$$

$$= 93\,845\text{ m}^2$$

$$\left|\Delta\vec{d}\right| = 306\text{ m}$$

$$\theta = \tan^{-1}\frac{71\text{ m}}{298\text{ m}}$$

$$= \tan^{-1} 0.238$$

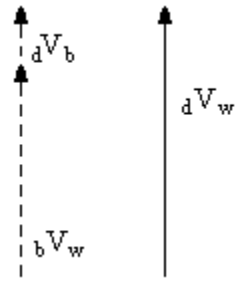
$$= 13°$$

$\Delta\vec{d} = 306\text{ m [N13°W]}$

REF: K/U OBJ: 1.1 LOC: FM1.02 KEY: FOP 3.11, p.122
MSC: P

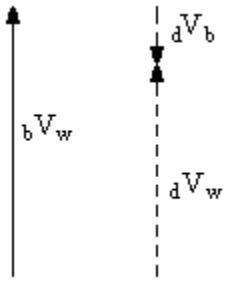
22. ANS:

(a)



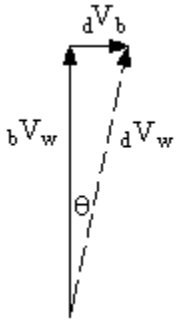
$$_d\vec{v}_w = {}_d\vec{v}_b + {}_b\vec{v}_w$$
$$= 1.6 \text{ m/s [N]} + 7.6 \text{ m/s [N]}$$
$$= 9.2 \text{ m/s [N]}$$

(b)



$$_d\vec{v}_w = {}_d\vec{v}_b + {}_b\vec{v}_w$$
$$= 1.6 \text{ m/s [S]} + 7.6 \text{ m/s [N]}$$
$$= -1.6 \text{ m/s [N]} + 7.6 \text{ m/s [N]}$$
$$= 6.0 \text{ m/s [N]}$$

(c)



$$\left|{}_{d}\vec{v}_{w}\right|^{2} = \left|{}_{d}\vec{v}_{b}\right|^{2} + \left|{}_{b}\vec{v}_{w}\right|^{2}$$

$$= (1.6 \text{ m/s})^{2} + (7.6 \text{ m/s})^{2}$$

$$= 60.32 \text{ (m/s)}^{2}$$

$$\left|{}_{d}\vec{v}_{w}\right| = 7.8 \text{ m/s}$$

$$\theta = \tan^{-1}\frac{1.6 \text{ m/s}}{7.6 \text{ m/s}}$$

$$= \tan^{-1} 0.2105$$

$$= 12°$$

${}_{d}\vec{v}_{w} = 7.8 \text{ m/s [N12°W]}$

REF: K/U OBJ: 1.5 LOC: FM1.02 KEY: FOP 3.11, p.123
MSC: P

23. ANS:

(a) The tip of the second hand makes one complete revolution in 60 s.

$$v_{av} = \frac{\Delta d}{\Delta t}$$

$$= \frac{2\pi (12 \text{ cm})}{60 \text{ s}}$$

$$= 1.3 \text{ cm/s}$$

(b) Since the speed is constant, $|\vec{v}| = v = 1.3$ cm/s
Therefore, $\vec{v}_{6} = 1.3$ cm/s [left]
$\vec{v}_{9} = 1.3$ cm/s [up]

(c) The displacement of the tip of the second hand in moving from the 3 to the 12 may be found from the following diagram:

Mathematical Solution:

$$\left|\Delta\vec{d}\right|^{2} = (12 \text{ cm})^{2} + (12 \text{ cm})^{2}$$

$$= 288 \text{ cm}^{2}$$

$$\left|\Delta\vec{d}\right| = 17 \text{ cm}$$

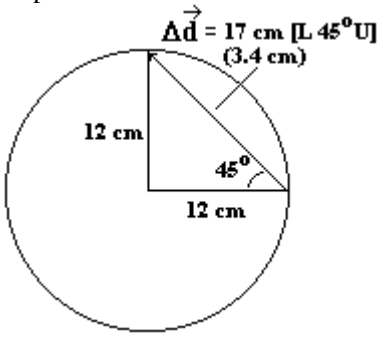
$\Delta\vec{d} = 17$ cm [left 45°up]

$$\vec{v}_{av} = \frac{\Delta \vec{d}}{\Delta t}$$

$$= \frac{17 \text{ cm [left 45°up]}}{45 \text{ s}}$$

$$= 0.38 \text{ cm/s [left 4°up]}$$

Graphical Solution:



REF: K/U OBJ: 1.1 LOC: FM1.02 KEY: FOP 3.3, p.92
MSC: SP