Acceleration (1-D and 2-D)

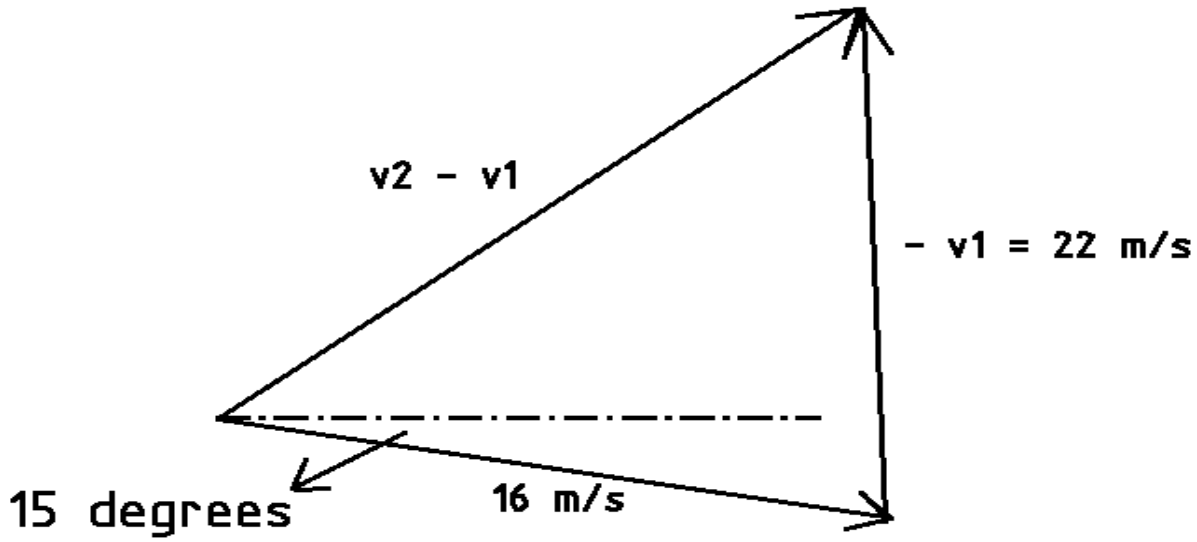
Acceleration ($\vec{a}$)

- The rate of change in velocity
- found by
  - taking the slope of a straight line v-t graph (uniform acceleration) or
  - the slope of the tangent to a curved v-t graph (instantaneous acceleration)
  - taking the slope of the line segment joining two points on a curved v-t graph (average acceleration)
  - using the equation $\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$
- is a vector quantity, so in 2-dimensions a vector diagram (and trigonometry) is required for addition/subtraction.

Example 1.

Selma is driving her Microbus™ directly south at 22 m/s when she makes a turn to a new heading of [E 15° S] with a new speed of 16 m/s. If the turn takes 7.2 s, what is her average acceleration?

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

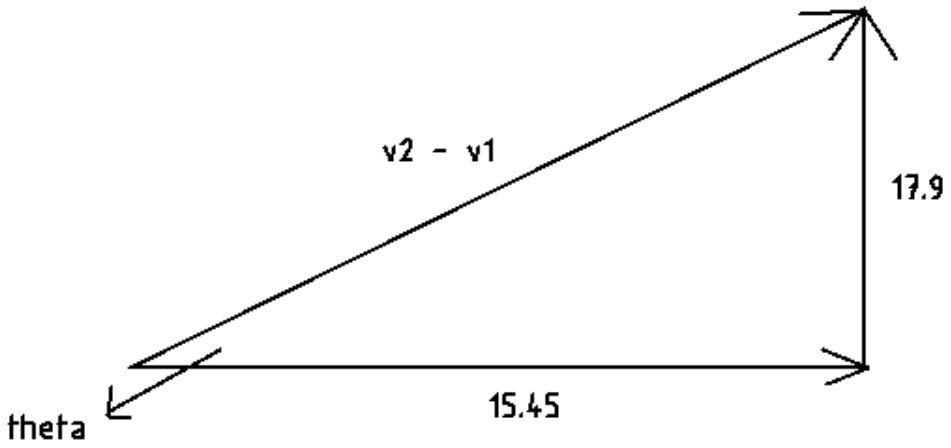


$\Delta v_x = 16 \cos 15°$

$= 15.45$ m/s

$\Delta v_y = 22 - 16 \sin 15°$

$= 17.9$ m/s



$$\Delta v = \sqrt{(17.9)^2 + (15.45)^2}$$

= 23.6 m/s

Tan θ = 17.9/15.45 θ = 49°

so

$$\vec{a} = \frac{23.6}{7.2}$$

= 3.3 m/s^2 [E 49° N]

Free Fall

- Any object that is only under the influence of gravity falls with constant acceleration (9.8 m/s^2 on Earth)
- Includes objects that are dropped, kicked, thrown or tossed, as long as they are in mid-air.
- Galileo first identified this constant acceleration.

We use the five equations of motion to solve free fall problems:

$$\vec{v}_2 = \vec{v}_1 + \vec{a}\,\Delta t \qquad \vec{\Delta} d = \vec{v}_1 \Delta t + \frac{1}{2}\vec{a}\,\Delta t^2$$

$$\vec{\Delta} d = \left(\frac{\vec{v}_1 + \vec{v}_2}{2} \right) \Delta t \qquad \vec{\Delta} d = \vec{v}_2 \Delta t - \frac{1}{2}\vec{a}\,\Delta t^2$$

$$\vec{v}_2{}^2 = \vec{v}_1{}^2 + 2\vec{a}\,\vec{\Delta} d$$

Example: Iggy throws a baseball straight up from his 5.5 m high balcony with a speed of 22 m/s.

a) What is the speed of the ball when it strikes the ground?
b) How long after he throws it does it reach the ground?
c) What is the ball's maximum height?

a) $\vec{v}_2{}^2 = \vec{v}_1{}^2 + 2\vec{a}\,\vec{\Delta} d$

$= (22)^2 + 2(-9.8)(-5.5)$

$v_2 = 24.3$ m/s

b) $\vec{v}_2 = \vec{v}_1 + \vec{a}\,\Delta t$

$-24.3 = 22 + (-9.8)(\Delta t)$

$\Delta t = 4.73$ s

c) $\vec{v}_2^{\,2} = \vec{v}_1^{\,2} + 2\vec{a}\vec{\Delta} d$

$0 = (22)^2 + 2(-9.8)(\Delta d)$

$\Delta d = 24.7$ m

so the maximum height of the ball is
(24.7 + 5.5) = 30.2 m [above the ground].