Centripetal Acceleration and Centripetal Force

From Yesterday

- The magnitude of centripetal acceleration is:

$$a_c = \frac{v^2}{r}$$

- Where v is the speed of the object and r is the radius of the arc or circle.
- Centripetal acceleration always points towards the center of the circle that the object is travelling on

Forces in Circular Motion

- The force that causes centripetal acceleration is a **net force** for objects in circular motion
- Using Newton's second law

$$\sum F = ma$$

- And the equation for centripetal acceleration we get:

$$\sum F = \frac{mv^2}{r}$$

- Example:
- A pilot starts to pull his plane (2100kg) out of a vertical dive maintaining a constant speed into an arc with a 1km radius causing a centripetal acceleration of 48.4 m/s^2
- What is the speed of the plane?
- What is the lift force of the plane? (perpendicular to the direction of the velocity)

For the speed of the plane

$$a_c = \frac{v^2}{r}$$

$$v^2 = ra_c$$

$$v = \pm\sqrt{ra}_c$$

$$v = \pm\sqrt{1000(48.4)}$$

Given:

$$a_c = 48.4 m/s^2$$

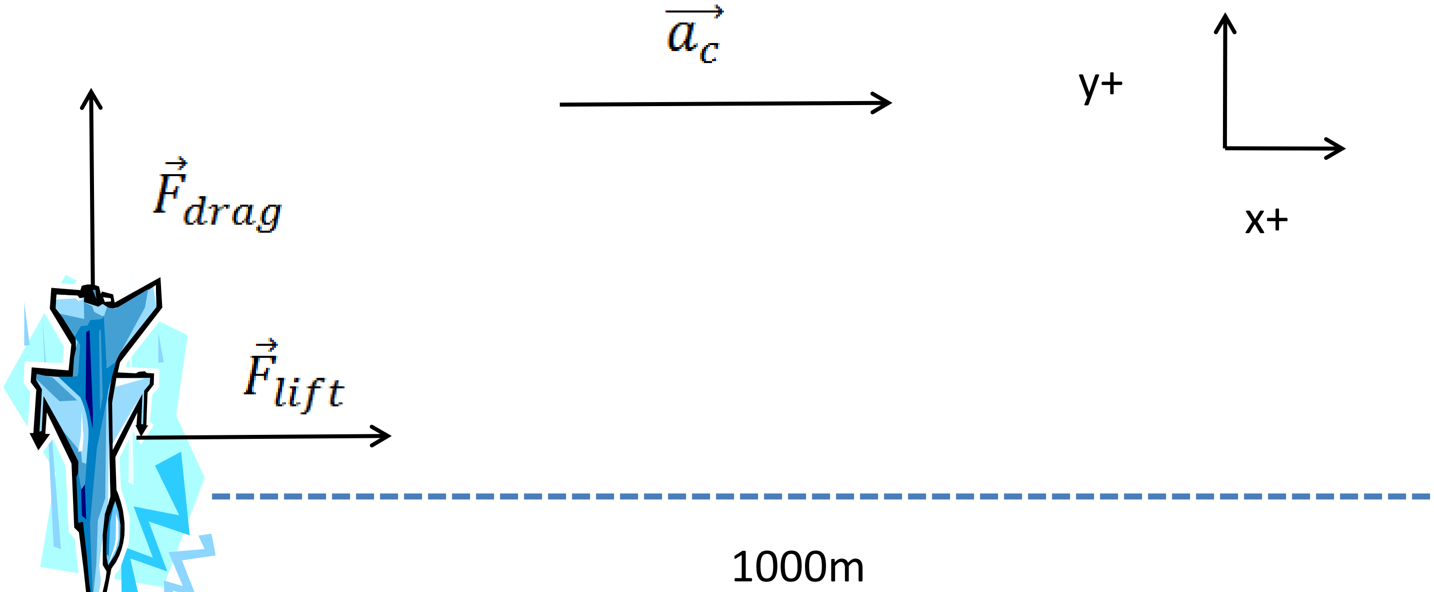
$$g = 9.8\ m/s^2$$

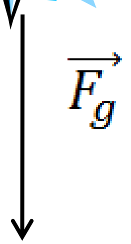
$$m = 2100\ kg$$

We will use the positive root because we are finding the magnitude of the velocity

$$v = 220\ m/s$$

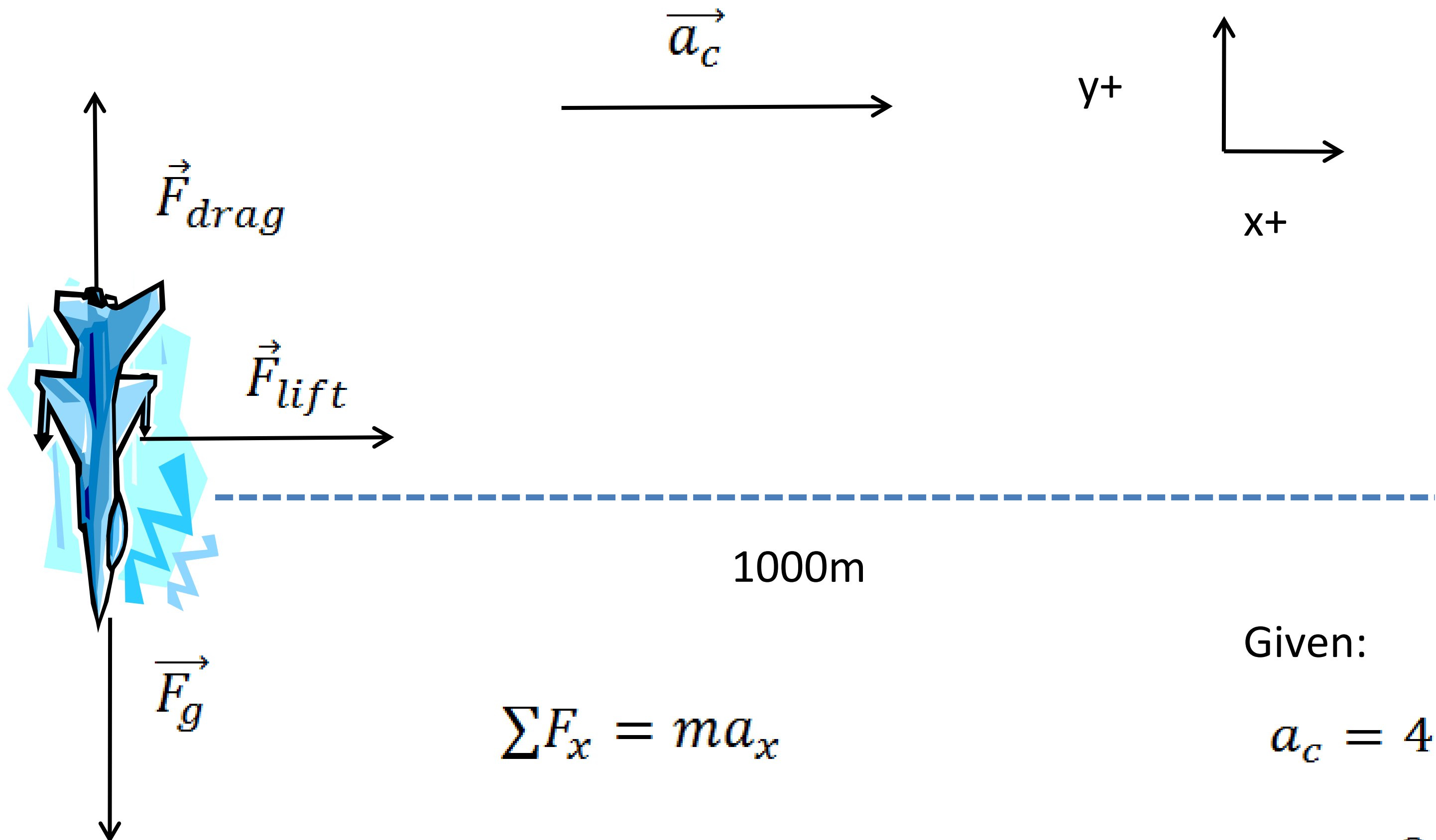
To find the lift force start by drawing a FBD of the plane





The forces acting on plane in the y direction are the force of gravity, and since we are assuming that there is no acceleration in this direction there must be a drag force as well.

The force in the x direction causing the centripetal acceleration is the lift force.



Given:

$$a_c = 48.4 m/s^2$$

$$g = 9.8\ m/s^2$$

$$m = 2100\ kg$$

$$\sum F_x = ma_x$$

$$F_{lift} = ma_c$$

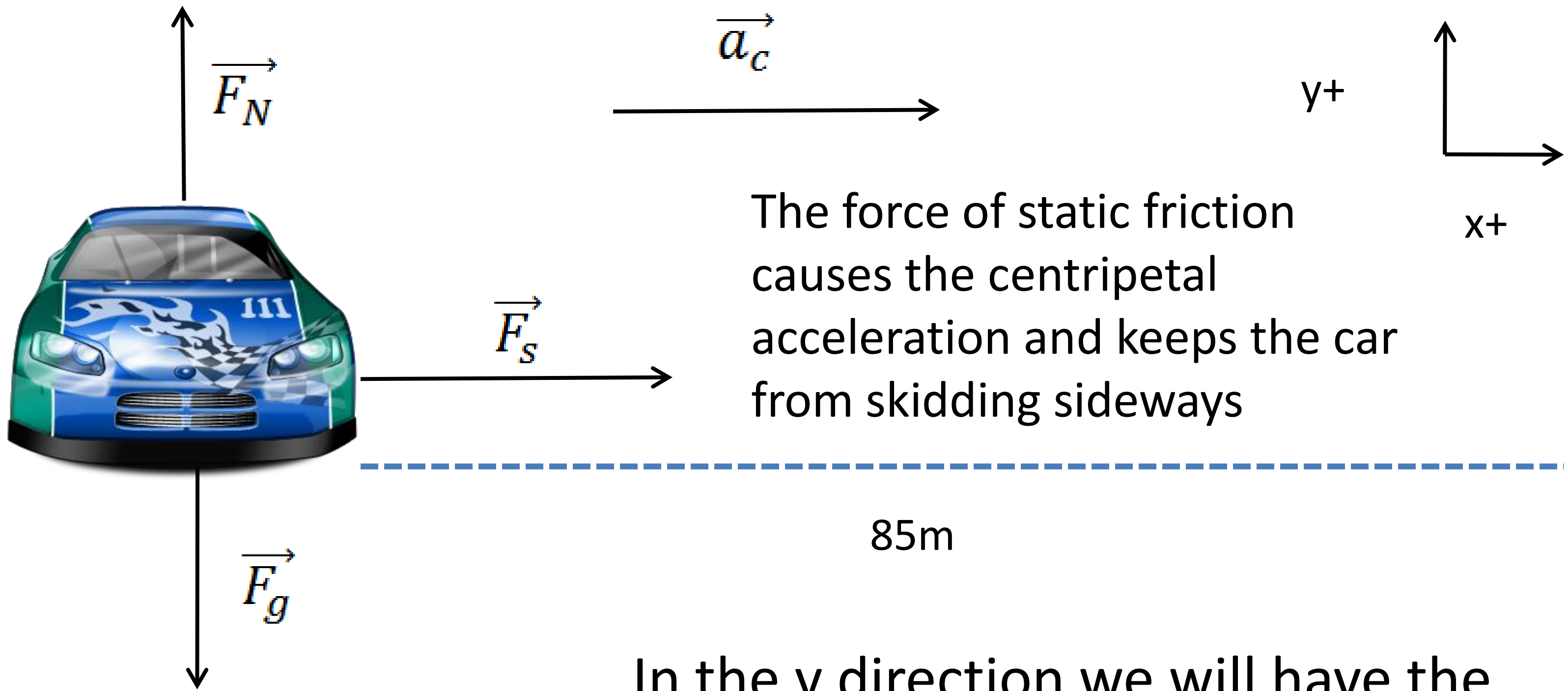
$$F_{lift} = 2100(48.4)$$

$$F_{lift} = 1.02 \text{ x } 10^5 N$$

Example:

What force is causing the centripetal acceleration of a car of mass 1200 kg going around a level corner of radius 85 m with a constant speed of 20 m/s. What is the magnitude of this force?

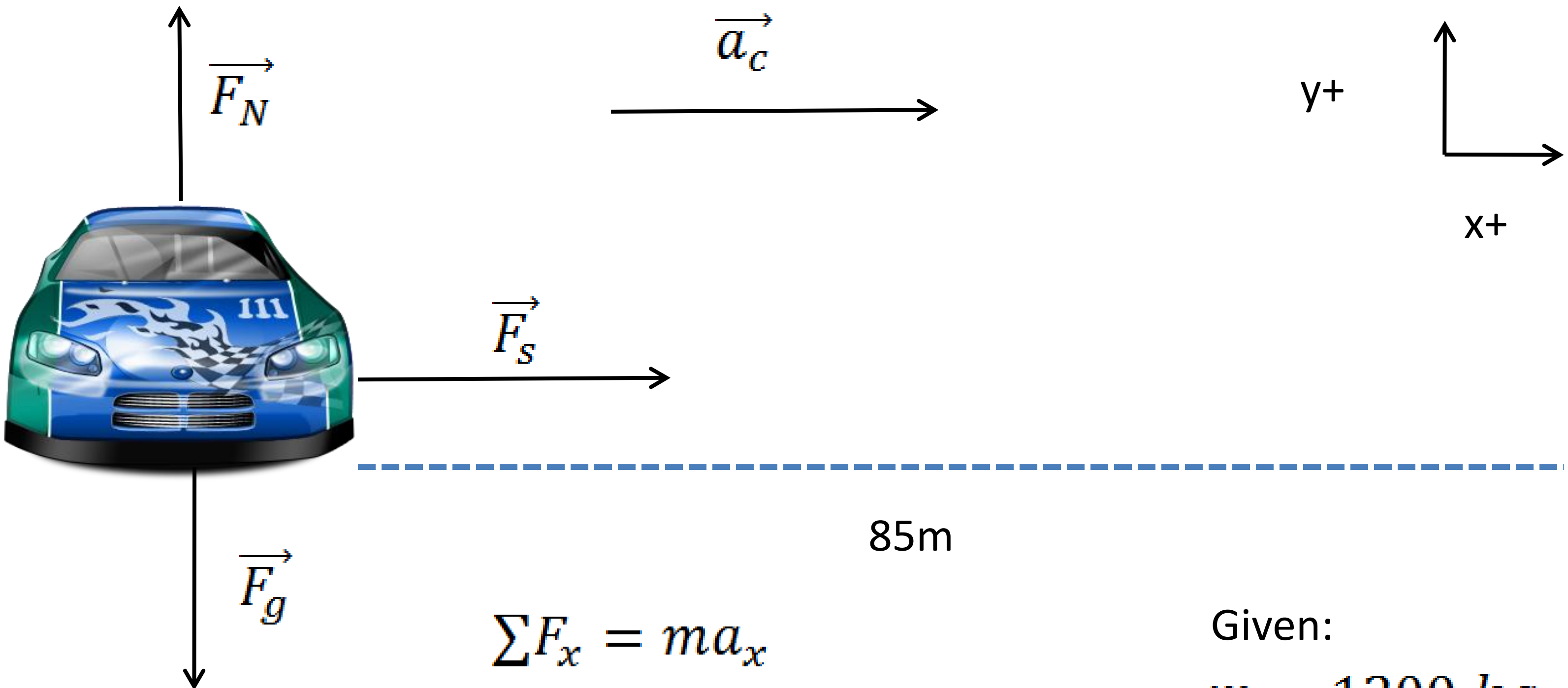
- Start with a free body diagram of the car



The force of static friction causes the centripetal acceleration and keeps the car from skidding sideways

In the y direction we will have the force of gravity and the normal force.

What force will be acting in the x-direction to cause the centripetal acceleration?



$$\sum F_x = ma_x$$

$$F_s = ma_c$$

$$F_s = m\frac{v^2}{r}$$

$$F_s = 1200\frac{(20)^2}{85}$$

$$F_s = 5647\ N$$

Given:

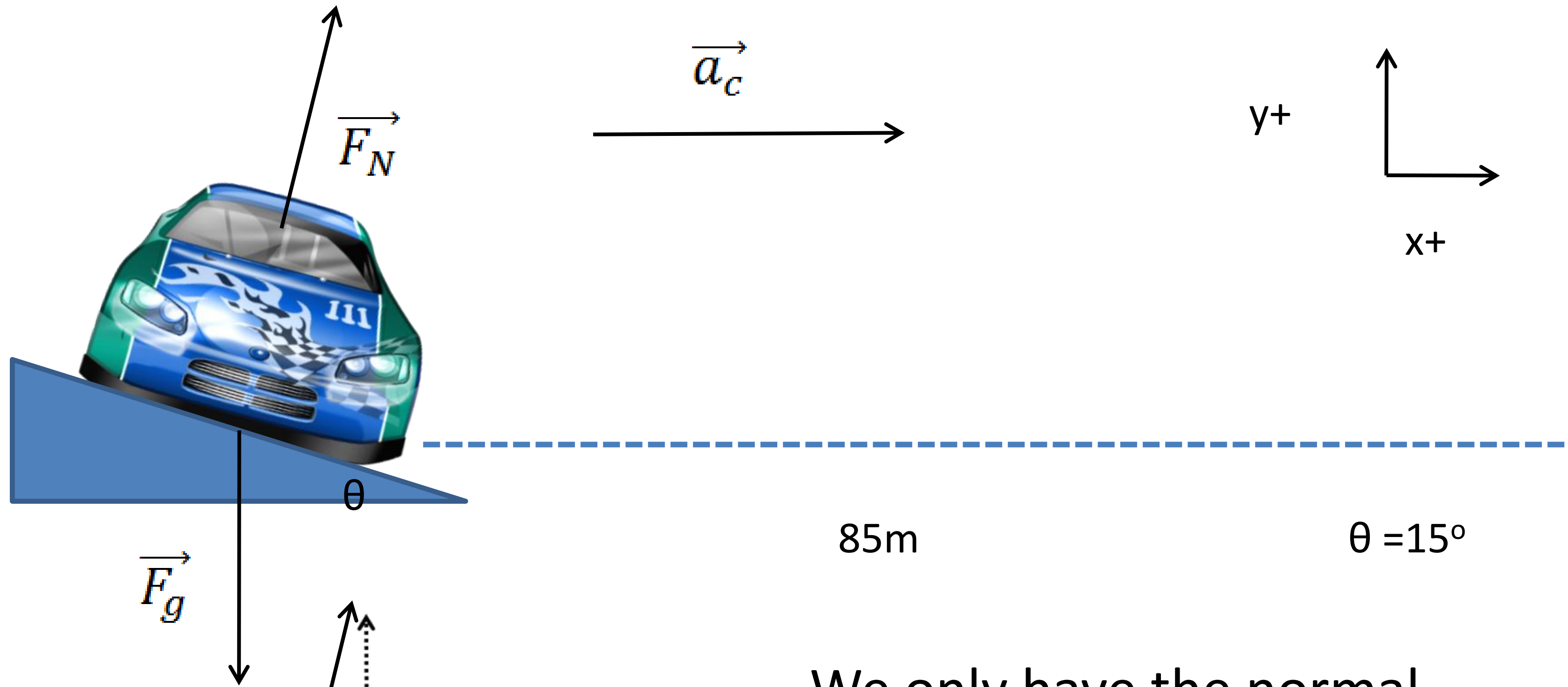
$$m = 1200\ kg$$

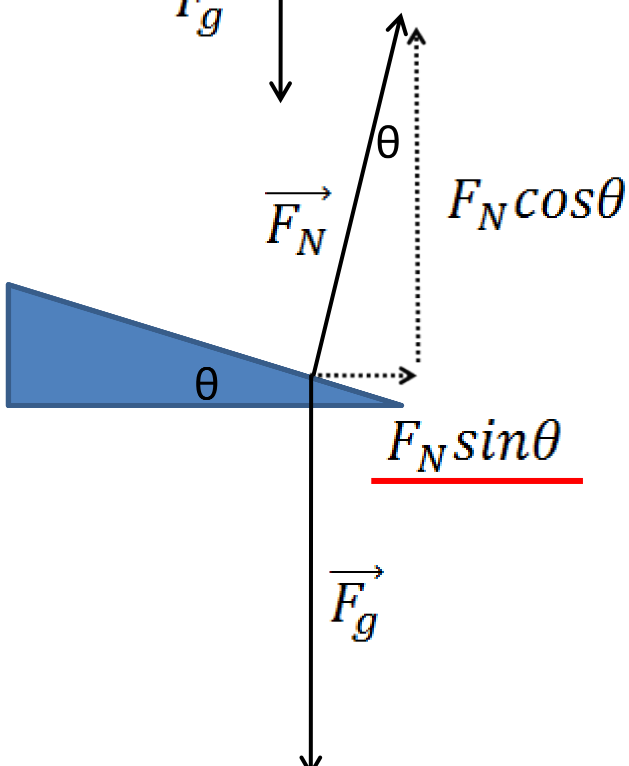
$$v = 20\ m/s$$

$$r = 85m$$

$$g = 9.8\ m/s^2$$

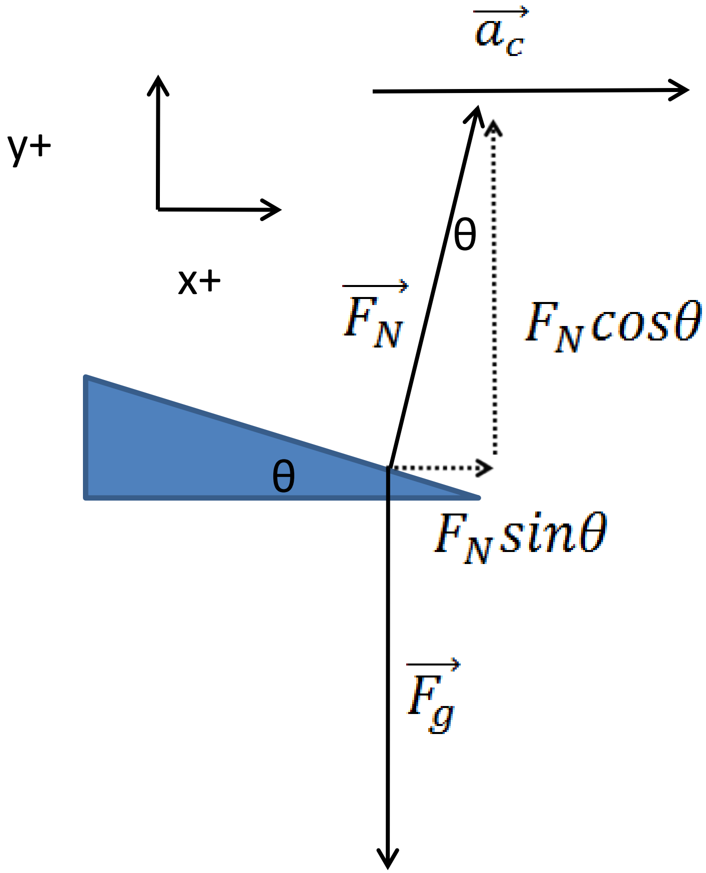
- Now consider the same car (1200 kg) going around a frictionless banked curve of the same radius (85 m). The angle of the bank is 15 degrees.
- What force is providing the centripetal acceleration?
- What speed must the car maintain to travel safely around the curve?





We only have the normal force and the force of gravity in this case

The component of the normal acting in the x direction will cause the centripetal acceleration



What speed must the car maintain to travel safely around the curve?

Look at the y –direction first

$$\sum F_y = 0$$

$$F_N cos\theta - F_g = 0$$

$$F_N cos\theta = mg$$

$$F_N = \frac{mg}{cos\theta}$$

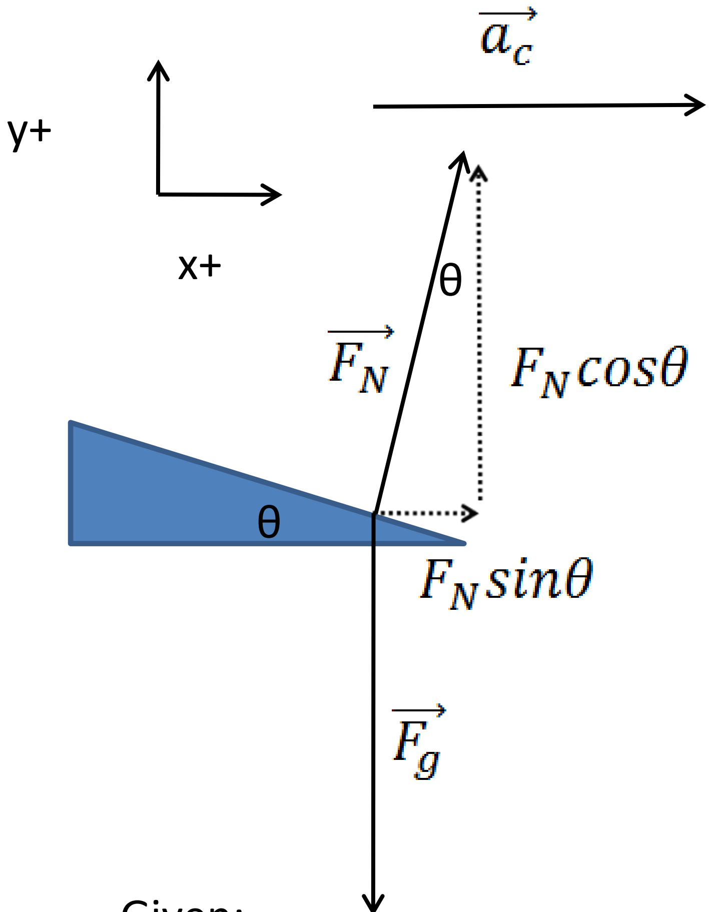
Given:

$m = 1200\ kg$

$r = 85m$

$\theta = 15^o$

$g = 9.8\ m/s^2$



Given:

$$m = 1200\ kg$$

$$r = 85m$$

$$\theta = 15^o$$

$$g = 9.8\ m/s^2$$

Now the x direction

$$\sum F_x = ma_x$$

$$F_N sin\theta = ma_c$$

$$\frac{mgsin\theta}{cos\theta} = ma_c$$

$$mgtan\theta = \frac{mv^2}{r}$$

$$v^2 = grtan\theta$$

$$v = \pm\sqrt{grtan\theta}$$

$$v = \pm\sqrt{9.8(85)tan15}$$

Now sub in

$$F_N = \frac{mg}{cos\theta}$$

We will use the positive root because we are finding the magnitude of the velocity

$$v = 14.94\ m/s$$

- If the car moves faster than 14.94 m/s it will slide up the banking, if it moves slower than 14.94 m/s it will slide downward.