

Using Phet's Quantum Wave Interference as an Interactive Lecture Demonstration

The Quantum Wave Interference simulation is a fantastic resource for making quantum mechanics tangible to students. The point of this particular lesson is to take students step by step through a series of 'experiments' that confront them with the basic conflict between the wave model and particle model. I like to do this with electrons rather than light because the electrons are more obviously particles than photons are. The demonstration shows their wavelike behaviour.

Go to http://phet.colorado.edu/new/simulations/sims.php?sim=Quantum_Wave_Interference.

Hit the **Run Now** button. Set up the simulation (using controls from top to bottom):

- high intensity
- screen: hits, medium brightness, no fade
- particles: electrons
- slits: 1/10 width, 1/2 separation, 3/4 vertical (this produces three clear maxima)
- add a potential barrier to block the right slit
- gun: 1/2 intensity (middle of slider), 1/2 velocity (default)

Note: the simulation shows the probability wave – it would be nice if it could be turned off. Don't draw attention to the wave. If students ask about it, praise them for noticing it and say that it will be covered later.

The rest of this file is similar to the four pages that the student gets, but with extra information and the answers to the questions in **bold**. As you go through the concept questions make the students write down an answer and a short reason. Then have them discuss their answers with their nearest neighbours. Then you can have a full-class discussion or go straight to the simulation. After they see what happens you can help them consider what the results mean. Give them time to write down their new ideas. This process has been shown to help students form the mental models needed in physics. The last four to six questions can be assigned for homework.

1) An electron gun is set to fire a couple hundred electrons per second. The screen at the back detects the electrons that make it through the slit(s).

The right slit is blocked off. If the gun is fired for two seconds you will see about 200 electrons

- a) in a narrow region opposite the slit
- b) mostly on the left half of the screen**
- c) only on the left half of the screen
- d) spread evenly over the whole screen



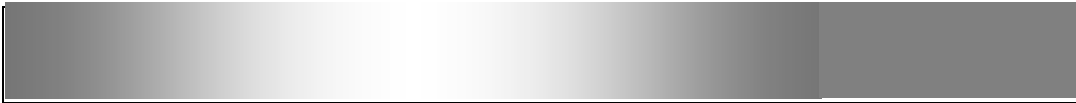
After the predictions are made, hit the button on the gun and hit it again two seconds later.

The shading above shows the average distribution because it was easier for me to do this on the computer. However, students should draw the dots to emphasize the randomness.

Copy the screen using the copy screen option so it can be compare to later results.

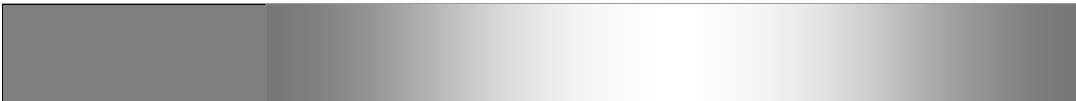
2) If the experiment is repeated the screen will

- a) exactly the same
- b) almost the same**
- c) very different



Clear the screen using the button to the right of the screen. Fire electrons for two seconds. Copy the screen. The two screens should be similar, but not identical.

3) The right slit is opened and the left slit is blocked. Sketch the pattern that will form.



How does this pattern compare to the previous one?

Clear the screen. Move the potential barrier to the left slit. Fire electrons. Copy the screen. The pattern is similar to the first two, but reflected about the middle of the screen.

4) Both slits are uncovered. The screen will show about 400 electrons that hit

- a) evenly across the screen
- b) in one concentrated region fading to the edges
- c) in two concentrated regions. The middle will have few hits.
- d) in three concentrated regions. The middle will have many hits.**



How does this pattern compare to the two single slit patterns?

Clear the screen. Clear the barrier. Fire for two seconds. Copy the screen.

The pattern for two slits is not a sum of the patterns from the two individual slits! Draw attention to the nodal regions. With two slits open, they receive fewer electrons than with one open. Adding electrons has resulted in fewer electrons in these places.

What does it look like? It is like the interference pattern you would get from water/light/sound waves passing through two slits.

What could be interfering? It could be electrons jostling each other, with ones that pass through the left slit bumping into those on the right slit. That's what water molecules do.

Suggest looking at the pattern when there is only one electron in the device at a time. Turn the intensity to almost zero. You should see an electron arrive only once or twice a second.

5) The intensity is reduced so that there will only be one electron going through the slits at a time. The next electron will land

- a) anywhere on the screen
- b) almost anywhere but most likely in the middle third
- c) **most likely in the middle or near 0.6 nm or 3.6 nm**
- d) most likely near 1.2 nm or 3.0 nm

Turn the gun on and leave it running while you have the students argue about this. It will take quite a while before the result is obvious.

Add a ruler and divide the screen into parts and assign the parts to particular groups. They can cheer when an electron lands in their spot. Make sure that it isn't fair, i.e. some groups get a region with nodes and others get a maximum. Eventually, they will see that the pattern is random but not even.

This shows a fundamental principle in quantum mechanics.

A) Intrinsic Randomness: We cannot know what will happen in a specific instant, but we can have a very good idea of what will happen if it is repeated many times. We have to give up causes and effects.

6) When the intensity of the gun is altered so that only one electron is travelling at a time. What will the pattern look like? You will see

- a) one spread out region that is the sum of the two spread out regions from each slit
- b) **three spread out regions because an individual electron acts like a wave**
- c) three spread out regions because the electrons ricochet off the slit edges

Once the simulation has been running long enough, they see that that the answer is either b or c. However, if it were due to ricocheting, the pattern should be a sum of the two individual ones. Draw attention to the nodal regions again. Suggest looking more closely to see which slit it went through.

7) A detector is added to the left slit. This will be able to detect whether the electrons went through the left side or the right slit. It will not block the electrons.

Sketch the Result:

Do

How does this pattern compare to the two single slit patterns?

Turn the gun up a bit to speed things along.

The pattern has changed! It is the sum of the two individual patterns, the pattern that you would suspect from particles fired through two slits.

The simulation doesn't let us record which slit it went through, nor does it tell us how the detecting is done. They have to believe you when you say that no matter how it is done, no matter how careful you are, once you can tell which slit they went through they behave like particles and there is no interference.

This experiment shows two key points, common throughout quantum mechanics, not just the two-slit experiment.

- B) Measurements Affect Reality: What you measure fundamentally changes the results of your measurement. No matter how gently and carefully you observe the electron - you affect it.**
- C) Superposition: If we don't know what state an object is in, then it is in a combination or superposition of those states and these possibilities can interfere with each other. If there are two indistinguishable paths - their probabilities can interfere. The electron can be in more than one place at a time. It can go through both slits.**

- 8) If you turn the detector off then electron goes through
- a) the left slit
 - b) the right slit
 - c) either the left or right, but we can't know which
 - d) both slits
 - e) neither slit

This pattern is a result of the interference of something that passes through both slits but the interpretation of this is debatable.

The electron is always detected as a whole electron. However, when it is not being detected its location is probabilistic and the probabilities are described extremely precisely by the Schrodinger Wave Equation. The electron's probability wave – not a smeared out electron - went through both slits and interfered with itself. The fuzzy blob coming out of the gun represents this probability wave as we imagine it. However, it can never be directly observed. What we observe is either an electron or no electron. The interpretation of what happens between the gun and the screen is still under debate. The main explanations are Copenhagen, many-worlds, sum over paths (Feynman) and the Bohm wave guide. Most physicists don't bother themselves with this, they just accept that QM works and get on with their work.

- 9) The wavelength of an electron is given by $\lambda = h/mv$. If the electrons move faster, then the
- a) pattern will get brighter
 - b) pattern will get fainter
 - c) **bright spots will move closer together**
 - d) bright spots will move further apart

This is not easy to see with the simulation. Set the gun on maximum intensity and the lowest speed. Slowly increase the speed and watch the probability wave, not the pattern for the clearest result.

At the lowest speeds you can only see three maxima. As you increase the speed, the pattern spreads less and you can see the next maxima as well.

The wavelength is smaller, therefore the pattern is spread out less. This is like blue light vs. red light. $x = \lambda L/d$.

- 10) How will the pattern for just a single slit change as the slit width is decreased? It will become
- a) fainter all over but the same size
 - b) fainter all over and narrower
 - c) **fainter directly opposite the slit**
 - d) fainter where it diffracts around the slit

Set the gun on maximum intensity and medium speed. Cover the right slit with a potential barrier. Look at the probability wave not the pattern on the screen for the clearest result.

As the slits get narrow, the intense direct wave gets reduced, but the edge effects remain. By removing most of the direct wave (which behaves most like a particle), the diffraction effects (wave-like behaviour) are more noticeable.

This is an example of another key aspect of quantum mechanics.

D) Heisenberg's Uncertainty Principle: They more you know about where the electron is (narrow slit) the less well you know its momentum (where it goes after the slit); $\Delta x \Delta p < h/2\pi$. This is a fundamental limit (like faster than light travel) and not a restriction of our technical abilities (like faster than sound travel was a century ago).

Show the video clip of a real electron experiment from Japan. So far, they have had to accept very weird results from a programmed simulation. It is important to show the real thing.

<http://www.hqrd.hitachi.co.jp/em/doubleslit.cfm> Click on movie clip 1. It is a short video - lasting only a minute. The students can examine the details later as homework.

- 11) You can't detect a person's wavelength because
- a) people are not waves, that's only for sub microscopic particles
 - b) the wavelength is ridiculously big
 - c) **the wavelength is ridiculously small**

A person's momentum will be much, much more than an electron's and so their wavelengths will be proportionately smaller.

- 12) A person's mass is on the order of 100 kg. Typical human speeds are on the order of 10^0 m/s and h is on the order of 10^{-33} Js. A human's wavelength is similar to the size of a
- a) cell
 - b) atom
 - c) nucleus
 - d) proton
 - e) **something else**

The wavelength is on the order of 10^{-33} m. The objects aren't nearly small enough. Cells are measured in micrometres (10^{-6} m), atoms are nanometres (10^{-9} m), nuclei in picometres (10^{-12} m) and protons are femptometres (10^{-15} m). The human wavelength, is close to the Planck length, which is the length that

results from combining h , G and c so they produce units in m . The length is given by the square root of hG/c^3 . and is $10^{-35} m$.

We can't show people interference but we can show very large molecules interfering. Show data of buckyball (buckminsterfullerene, C60) interference.

<http://www.quantum.univie.ac.at/research/matterwave/c60/index.html>

Go down one page to show how the shape resembles a soccer ball. The next page shows a photo of the lab and equipment. The next page shows the results with and without the grating. Point out the extra two maxima with the grating.

13) Go to the simulation at

http://phet.colorado.edu/new/simulations/sims.php?sim=Quantum_Wave_Interference. Select helium atoms as your particle. On the screen settings choose 'hits', not average intensity and disable the fade option. Add double slits.

a) At the default settings, how far is the first maximum from the center? Use the ruler.

Around 1.1 nm.

b) What are the slit separations and speed of the helium at this setting?

1.2 nm, 0.15 km/s

c) What is the wavelength of helium at this speed?

$$\lambda = h/mv = 6.63 \times 10^{-34} \text{ Js} / 4.00 \times 1.66 \times 10^{-27} \text{ kg} \times 150 \text{ m/s} = 0.67 \text{ nm}$$

d) Calculate how far the slits must be from the screen?

$L = x_1 d/\lambda = 1.1 \text{ nm} \times 1.2 \text{ nm}/0.67 \text{ nm} = 2.0 \text{ nm}$. This agrees with the measurement on the simulation.

e) Alter the settings until you get the clearest pattern of central three maxima. Describe in detail what you must do and why to achieve this.

Make the slits as narrow as possible to increase the size of the nodes, so the maxima are more distinct. Make the slit separation about twice the size of the slit widths. If d is too small, the central maximum dominates and it turns into a single slit pattern. If d is too large, the interference weakens and the central maximum starts to disappear. Increasing the vertical distance separates the maxima more, but they soon go off the screen, the middle setting is best. Similarly, decreasing the speed will increase the wavelength and spread the pattern more, but the middle setting is best.

14) The real electron experiment was performed by A Tonomura in 1989. The electrons were accelerated by a potential difference of 5×10^4 m/s V. Do the following calculations non-relativistically. (Details can be found at <http://www.hqrd.hitachi.co.jp/em/doubleslit.cfm>.)

a) How fast were these electrons moving?

Use kinetic energy equals electric energy and you get a speed of 1.3×10^8 m/s if calculated non-relativistically. $v = \sqrt{2qV/m} = \sqrt{2 \times 1.6 \times 10^{-19} \text{ C} \times 5 \times 10^4 \text{ V} / 9.11 \times 10^{-31} \text{ kg}}$. Site gives 1.2×10^8 m/s. To one digit these are the same.

b) How fast is that compared to the speed of light? 42 %.

Site says about 40%.

c) How long would they take to cross the 1 m apparatus?

$t = d/v = 7.9$ ns.

d) What was the wavelength of these electrons?

Use the De Broglie equation. $\lambda = h/p = 6.63 \times 10^{-34} \text{ Js} / (9.11 \times 10^{-31} \text{ kg} \times 1.3 \times 10^8 \text{ m/s}) = 5.6 \times 10^{-24} \text{ m}$.

e) Should relativistic equations be used?

Gamma is 1.1, so if we are only concerned with one digit there is no problem. For two digits we should use relativity.

f) How was the slit set-up in this experiment different from the simulation?

The slit was formed by an "electron biprism" ... "which consists of two parallel plates and a fine filament at the center. The filament is thinner than 1 micron (1/1000 mm) in diameter. " You can demonstrate something similar by putting a straight pin in the path of a laser pointer beam.

15) The experiment demonstrating interference of buckminsterfullerene, C₆₀, had the molecules moving at 210 m/s. Each molecule has an atomic mass of 720 atomic units and a diameter of 1 nm. The molecules passed through slits with widths of 50 nm and separations of 100 nm. After the slits, the molecules travelled 1.25 m before being detected. (More details can be found at <http://www.quantum.univie.ac.at/research/matterwave/c60/index.html>.)

a) What is the mass of one molecule?

$720 \times 1.660 \times 10^{-27} \text{ kg} = 1.195 \times 10^{-24} \text{ kg}$.

b) What is the momentum?

$1.195 \times 10^{-24} \text{ kg} \times 210 \text{ m/s} = 2.51 \times 10^{-22} \text{ kg m/s}$

c) What is its wavelength?

$h/p = 2.65 \times 10^{-12} \text{ m} = 0.00265 \text{ nm}$

d) How does this wavelength compare the size of the molecule?

1/500! The wavelength is much smaller than the object itself.

e) How does this wavelength compare the size of the slits?

1/50, 000! We often say that to demonstrate interference you need a wavelength comparable to the slit separation. What we really mean is that this is what is needed to demonstrate it easily.

f) How far apart would the slits be if you were demonstrating the interference of light with a ratio like this?

$d = 5 \times 10^{-7} \text{ m} \times 50,000 = 2.5 \text{ cm}$.

g) How far apart would the fringes be separated if the screen was 5 m from the slits?

If the screen was 5 m away the fringes would be separated by a tenth of a millimetre. This could be detected but not easily!

h) How far from the center would the first maximum be?

$x_1 = \lambda L/d = 2.65 \times 10^{-12} \text{ m} \times 1.25 \text{ m} / 1.00 \times 10^{-7} \text{ m} = 3.31 \times 10^{-5} \text{ m}$.

i) How was the slit set-up different from the simulated experiment?

A diffraction grating of many slits was used to intensify the interference.

16) The two-slit experiment illustrated several fundamental concepts in quantum mechanics. Explain what each of these is and how they are demonstrated in the experiments:

a) Intrinsic Randomness

We cannot know what will happen in a specific instant, but we can have a very good idea of what will happen if it is repeated many times. We have to give up causes and effects on an individual basis. We don't know where the next electron will land but we know what the pattern of 100 electrons will be.

b) Measurements Affect Reality

What you measure fundamentally changes the results of your measurement. No matter how gently and carefully you observe the electron - you affect it. A detector at one slit destroys the interference pattern of two slits.

c) Superposition

If we don't know what state an object is in, then it is in a combination or superposition of those states and these possibilities can interfere with each other. If we don't try to detect which slit the electron goes through then electron can be in more than one place at a time. The probability of where it goes is a result of its amplitude of going through both slits.

d) Heisenberg's Uncertainty Principle

The smaller the uncertainty in position - the greater the uncertainty in momentum - $\Delta x \Delta p < h/2\pi$. This is a fundamental limit (like faster than light travel) and not a restriction of our technical abilities (like faster than sound travel was a century ago). The better you know where the electron was (narrow slits) the less well you know its momentum (where it goes after the slit) and its position at the screen becomes less predictable.

17) Google "Dr. Quantum double slit". This should take you to a very popular and well-animated version of the electron two-slit experiment. Unfortunately it has the physics seriously wrong in a few places. Describe these.

The biggest error, is that the double slits appear to give a single slit interference pattern with the central maximum twice as wide as the others. The next most serious, is that when the electrons are sent through one slit they show almost no diffraction. If the diffraction patterns don't overlap, they can't interfere. It shows the electron smearing out and splitting into two, which can leave you imagining that this is what interferes. However, the interference pattern is due to the interference of probability not smeared out matter.